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VERIFICATION OF BRITTLE FRACTURE CRITERIA FOR BIMATERIAL STRUCTURES

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Abstract: The increasing application of composite materials in the construction of machines causes strong need for modelling and evaluating their strength. There are many well known hypotheses used for homogeneous materials subjected to monotone and cyclic loading conditions, which have been verified experimentally by various authors. These hypotheses should be verified also for composite materials. This paper provides experimental and theoretical results of such verifications for bimaterial structures with interfacial cracks. Three well known fracture hypotheses of: Griffith, McClintock and Novozhilov were chosen. The theoretical critical load values arising from each hypotheses were compared with the experimental data including uni and multi-axial loading conditions. All tests were carried out with using specially prepared specimens of steel and PMMA.

Key words: Fracture Criteria, Interfacial Crack, Delamination, Experimental Tests

1. INTRODUCTION

The development of composite structures and manufacturing technology allow the production of materials of specific strength and functional properties, which are more convenient for use in machine design. Selected in suitable proportions homogeneous components, when combined together, provide high stiffness and strength with a lower weight. In addition, reinforced with particles (molecular embedded in a matrix) composite materials exhibit better functional properties such as hardness, abrasion resistance and higher temperature work. Additionally, by using the reinforcement of the lower stiffness than the matrix, higher fracture toughness may be obtained (Wojciechowski et al. 1993). In many cases the properties of the composite material directly depend on the characteristics of the individual components.

Composite materials are characterized by macroscopic inhomogeneity of the structure where the local high stress gradients are produced by forced compatibility of displacements on the bond or caused by the local material discontinuities or sharp corners. This kind of stress raisers can generate singular stress fields of qualitative characteristics other than produced by the faults situated in a homogeneous material. Such mechanical fields can have multiple singularities described by real and complex exponents of particular terms when using asymptotic approach to the problem. These factors cause difficulties of strength estimations for such structures.

In many cases a theoretical method of homogenization of material properties may be useful (Woźniak 1987). Another approach to determining the strength of multiphase materials is based on the fracture parameters as generalized stress intensity factors related to the local stress field existing around the crack tips, can be found in the following papers: Salganik (1948), Williams (1959), Erdogan (1963), Rice and Sih (1965), Chen (1992) among others.

2. MAIN OBJECTIVES AND ASSUMPTIONS OF THE WORK

Most of the fracture mechanics hypotheses applied for estimating strength of homogeneous materials with stress raisers, such as sharp notches or cracks producing singular stress fields, have been verified experimentally by many authors, e.g. Murakami (1987), Seweryn and Łukaszewicz (2002). The correctness of their use in structural components made of composite materials with stress raisers producing qualitatively different stress fields, such as single or multiple "oscillatory" singularities, requires experimental verification.

The purpose of this work is to verify the three fracture strength hypotheses of Griffith, modified McClintock and Novozhilov, applied for predicting strength and fracture toughness of bi-material structures including interface cracks.

The present work is based on some general assumptions:

1. Both bonded materials can be considered as perfectly connected Hookean media.
2. Fracture process appears exclusively along the material interface. It means that fracture conditions are analyzed along the bond only and no such conditions are investigated separately for the homogeneous component.
3. There is no influence of the bonding material (glue) on the real stress distribution. Since the adhesive layer is very thin, instead of investigating real stress distribution we are focused on the predominant stress field produced by the “ideal” connection, which is a general base for calculating and comparing the critical parameters of the process. Similar assumption is usually made when analyzing fracture of a homogeneous material, where the influence of the small plastic zone is discarded, in spite of the fact that it may change the stress distribution at the vicinity of the crack tip.
4. Only cohesive damage mechanism is taken into account.

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These assumptions make it possible to use simplified mathematical apparatus for calculating basic critical parameters from the known formulas describing local stress fields along the bond.

3. FRACTURE STRENGTH CRITERIA

3.1. Griffith energy criterion

This criterion is based on the assumption that the brittle fracture occurs when the energy release rate G reaches the critical value: $G = G_c$. For a structural element with an interfacial crack, G parameter is expressed by formula (1) containing generalized stress intensity factors $K_j^{(\lambda r)}$ and material constants, as follows:

$$G = \frac{1}{16\cosh(\pi\delta)^2}\left(\frac{1+\kappa_1}{\mu_1}+\frac{1+\kappa_2}{\mu_2}\right)\left(K_I^{(\lambda_r)^2}+K_{II}^{(\lambda_r)^2}\right) \tag{1}$$

where:

$$\begin{aligned} K_I^{(\lambda_r)} &= \lim_{r\to 0}\sqrt{2\pi}r^{1-\lambda_r}\left(\sigma_\varphi(r,0)\cos(\Delta)+\tau_{r\varphi}(r,0)\sin(\Delta)\right) \\ K_{II}^{(\lambda_r)} &= \lim_{r\to 0}\sqrt{2\pi}r^{1-\lambda_r}\left(\tau_{r\varphi}(r,0)\cos(\Delta)-\sigma_\varphi(r,0)\sin(\Delta)\right) \end{aligned} \tag{2}$$

and μ_i – shear modulus, ν_i – Poisson's ratio, $\kappa_i = \frac{3-\nu_i}{1+\nu_i}$ for plane stress and $\kappa_i = 3-\nu_i$ for plane strain, λ_r – real part of the complex exponent, $\delta = \frac{1}{2\pi}\ln\left(\frac{\kappa_1\mu_2+\mu_1}{\kappa_2\mu_1+\mu_2}\right)$ – imaginary part of the complex exponent, $\Delta = \delta\ln(r)$.

3.2. McClintock criterion

McClintock (1958) proposed strain fracture condition, assuming that the fracture occurs when the normal strain ε_φ in a small distance ρ_c ahead of the crack tip reaches the critical value:

$$\varepsilon_\varphi(r=\rho_c) = \varepsilon_c \tag{3}$$

In practice, the stress form of this criterion proposed by Ritchie (1973) is frequently used, where strains have been replaced by the corresponding stress components. Crack propagation occurs when the hoop stress σ_φ at a finite distance $r = \rho_c$ reaches the critical value σ_c (Fig. 1). This condition can be written as:

$$\max_\varphi \sigma_\varphi\,(r=\rho_c) = \sigma_c \tag{4}$$

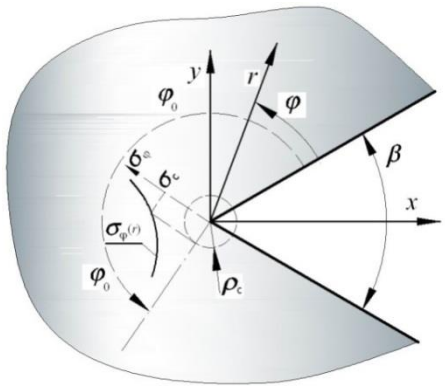

Fig. 1. Graphical interpretation of modyfied McClintock criterion

The characteristic parameter ρ_c is considered as a material constant and can be determined experimentally using formulas (5) and (6), where K_c and σ_c are known.

$$\frac{K_{Ic}}{\sqrt{2\pi\rho_c}} = \sigma_c \tag{5}$$

$$\rho_c = \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_c}\right)^2 \tag{6}$$

3.3. Novozhilov non-local stress criterion

Fracture criterion, proposed by Novozhilow (1969) and modified by Seweryn and Mróz (1998), assumes that the process of fracture occurs when the average stresses over a certain distance ahead of the notch root, represented by the stress function $R_\sigma(\sigma_n,\tau_n)$, reaches the critical value causing material decohesion (Fig. 2). This condition can be written as follows:

$$\max_{(\varphi)} \bar{R}_\sigma(\sigma_n,\tau_n) = \max_{(\varphi)}\left[\frac{1}{d_0}\int_0^{d_0} R_\sigma(\sigma_n,\tau_n)dr\right] = 1 \tag{7}$$

where: $\bar{R}_\sigma(\sigma_n,\tau_n)$ – non-local fracture function, $R_\sigma(\sigma_n,\tau_n)$ – local fracture function, σ_n, τ_n – normal and shear stress on the physical plane, r, φ – polar coordinates of reference system located at the top of the concentrator, d_0 – damage zone size.

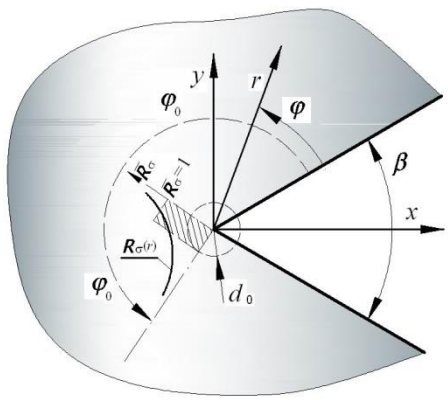

Fig. 2. Local fracture function R, averaged in a plane ahead of the apex

The local stress function $R_\sigma(\sigma_n,\tau_n)$ can be determined (depending on the type of the concentrator), for instance, in conditions of normal, equivalent or principal stresses. This criterion makes it possible to assess the critical load value as well as determine the direction of crack propagation, which corresponds to the maximum value of the nonlocal fracture function.

The value of the parameter d_0 can be determined from the known fracture parameters as K_c and σ_c, where the local fracture function is defined by a hoop stress $R_\sigma(\sigma_n,\tau_n) = \sigma_\varphi$.

$$\frac{1}{d_0}\int_0^{d_0}\frac{K_{Ic}}{\sqrt{2\pi r}}dr = \sigma_c \tag{8}$$

Thus, formula (9) provides solution for d_0:

$$d_0 = \frac{1}{2\pi}\left(\frac{2K_{Ic}}{\sigma_c}\right)^2 \tag{9}$$

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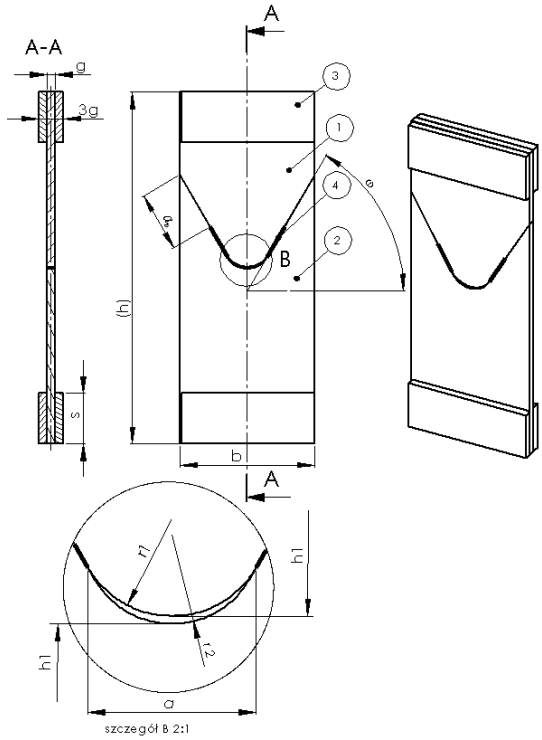
4. MATERIAL AND METHOD

4.1. Test specimens

The specimens used in the experimental studies are presented in Fig. 3. All samples have been prepared by connecting two different parts of steel and PMMA plates but the same thickness. Both parts were glued symmetrically using Loctite 401 and leaving three cracks – a central and two lateral interfacial cracks. Internal surfaces of the connections have been specially shaped to form a symmetrical arched crack in the middle of specimens, providing good fitting of the components and causing fracture along the interface.

All specimen dimensions are presented in Tab. 1.

a)



b)

Fig. 3. Bimaterial specimen with interfacial cracks: a) geometrical characteristics: 1 – S235JR steel, 2 – PMMA, 3 – straps, 4 – connection zone; b) SP45PL specimen ($\omega = -45^0$)

Tab. 1. Basic dimensions of the specimens with interfacial cracks

No.	Symbol	ω [°]	r1 [mm]	r2 [mm]	h [mm]
1	SP60PL	-60	16	17.4	251.4
2	SP45PL	-45	18	21.3	251.4
3	SP30PL	-30	22	30	251.2
4	SP0	0	120	120	196
5	SP30S	30	30	22	251.2
6	SP45S	45	21.3	18	251.4
7	SP60S	60	17.4	16	251.4
a [mm]=30, b [mm]=96, g [mm]=6, s [mm]=36, h1 [mm]=125					

For all specimens the gluing process was performed in similar conditions (compression, time etc.) by using a specially designed device. We also monitored temperature and humidity.

4.2. Testing procedure and damage mechanisms

All tests have been carried out on the hydraulic machine INSTRON 8502 equipped with additional proving ring positioned between the specimen and the upper holder of the machine. This dynamometer allowed to increase the accuracy of the load applied. In order to provide alignment of load the specimens have been mounted in a specially designed holder, shown in Fig. 4.

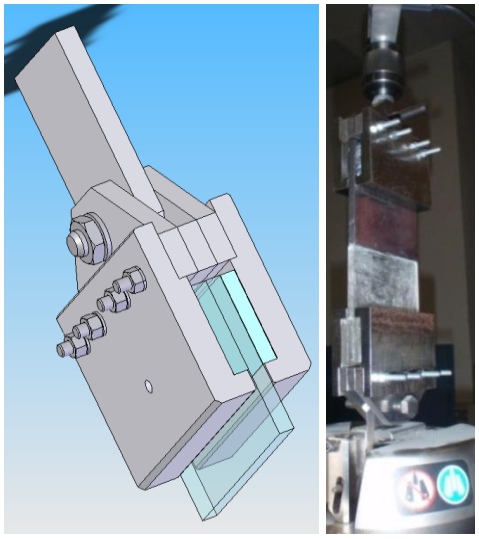


Fig. 4. Specially designed specimen holders used during testing

For all types of specimens fracture occurred along the interface. Next, it was necessary to analyse fracture surfaces in order to distinguish between a cohesive fracture and the adhesive damage. This was determined in two different ways. Firstly, all fractured specimens were valuated visually. In case of cohesive damage, a small rough layer was found on both surfaces – steel and PMMA. When adhesive failure occurred, the steel surface was clean and free from any particles.

Secondly, both damage mechanisms - adhesion and cohesion - were analysed by considering breaking force values. Critical forces for adhesive damage were significantly lower than these corresponding to cohesive fracture. So all results obtained for adhesive damage were omitted. The number of tested specimens

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of the same shape depended on the discrepancy of the critical force and the angle ω (Fig. 3), and varied from 7 to 13.

5. EXPERIMENTAL AND THEORETICAL RESULTS

Tab. 2. Critical values of stress σ_c and the stress intensity factor K_c

	Loctite 401
σ_c [MPa]	17
K_c [MPa m$^{1/2}$]	0.679

Tab. 3. Parameters defining the length of the damage zones

	McClintock ρ_c [mm]	Novozhilov d_0 [mm]
Loctite 401	0.254	0.404

Tab. 4. Critical forces Fc [N] obtained from experiments and calculated using various hypotheses

Specimen type	Critical forces Fc [N] determined experimentally	Criteria		
		Novozhilov	McClintock	Griffith
		Fc [N]	Fc [N]	Fc [N]
SP60PL	1423	1244 H 976 R 972 P	1584 H 1403 R 1339 P	1399
	1441			
	ΣFc$_i$/i=**1432**			
SP45PL	1056	1040 H 1058 R 969 P	1362 H 1493 R 1336 P	1097
	1063			
	ΣFc$_i$/i=**1059**			
SP30PL	1283	1169 H 1266 R 1137 P	1577 H 1663 R 1504 P	1283
	1199			
	1172			
	1254			
	ΣFc$_i$/i=**1227**			
SP0	1664	1570 H 1419 R 1349 P	2176 H 1791 R 1751 P	1621
	1696			
	1708			
	1728			
	ΣFc$_i$/i=**1699**			
SP30S	703	737 H 785 R 709 P	973 H 1091 R 970 P	714
	660			
	685			
	ΣFc$_i$/i=**683**			
SP45S	723	882 H 717R 705 P	1127 H 1030R 973 P	692
	678			
	742			
	607			
	ΣFc$_i$/i=**688**			
SP60S	927	1079 H 562 R 638 P	1319 H 797 R 863 P	891
	908			
	960			
	ΣFc$_i$/i=**932**			

Damage functions applied to fracture criteria: P – principal stress,
H – hoop stress normal to the fracture direction, R – reduced, Huber-von Mises-Hencky equivalent stress

The critical forces – obtained experimentally and calculated theoretically, using various hypotheses – are shown in Tab. 4. In order to calculate critical stress intensity factors, energy release rates G_c and critical forces obtained from investigated fracture strength hypotheses, the numerical *FEM* solutions have been performed and definitions of the stress intensity factors given by the formula (2) were applied.

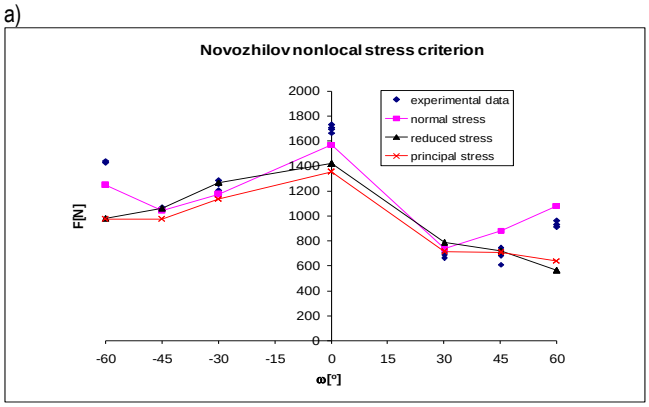
In order to calculate critical forces using various hypotheses some material parameters had to be determined. Due to the fact that crack propagation proceeded along the bond these parameters have been determined for glue used in experiments (Loctite 401). The critical values of the stress intensity factor K_c and the critical stresses σ_c for the bond have been obtained experimentally from fracturing two halves of the same material with and without a central crack. These critical parameters as well as damage zone sizes are given in Tab. 2 and Tab. 3.

Application of the hypotheses described in chapter 3 (except the Griffith criterion) requires analytical description of the stress field occurring around the tip of interfacial crack. The proper formulas are known from the literature e.g. by Erdogan (1963) and Molski and Mieczkowski (2004).

When testing Novozhilov and McClintock criteria, independent verification was performed using the maximum principal stress, the normal stress to the direction of fracture (hoop stress) and reduced equivalent stress by H-M-H hypothesis, as a function of damage. In the asymptotic solution there was taken into account only the first singular term.

Critical forces obtained from the experimental tests and hypotheses tested are shown in Tab. 4 and illustrated graphically in Fig.5 and 6.

a)



b)

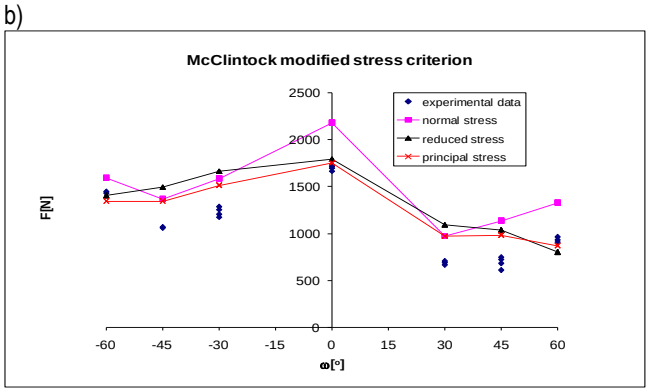


Fig. 5. Comparison of critical load values obtained from tests (a-Novozhilov, b-McClintock) with experimental data for different damage functions vs interfacial crack angle

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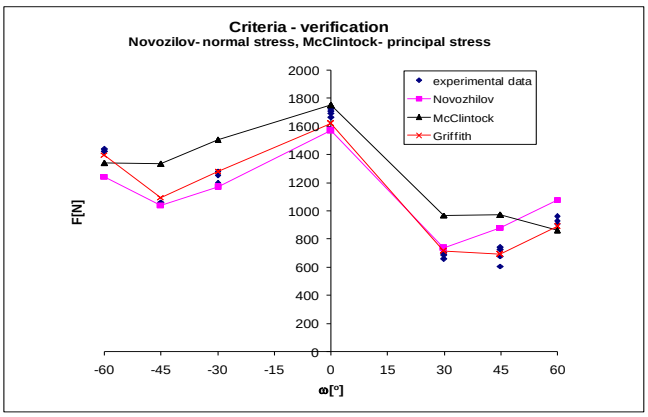

Fig. 6. Comparison of critical load vs. ω angel obtained from test and calculated using various criteria

6. CONCLUSIONS

In the comparison of critical forces obtained from the experimental tests with the evaluated theoretical data leads to the following conclusions:

– the most accurate results were obtained using the Griffith criterion taking into account proportions between the normal and tangential stresses at the interface;
– for the Novozhilov criterion, most accurate results were obtained for the normal hoop stresses considered as a damage function;
– for the McClintock criterion - the maximum principal stresses considered as a damage function gave the most accurate values of the critical force.

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