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THE FORMATION AND PROPAGATION OF SOLITON WAVE PROFILES FOR THE SHYNARAY-IIA EQUATION

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Abstract: This study examines a novel use of the Jacobi elliptic function expansion method to solve the Shynaray-IIA equation, a significant nonlinear partial differential equation that arises in optical fiber, plasma physics, surface symmetry geometry, and many other mathematical physics domains. This kind of solution has never been attained in research prior to this study. Numerous properties of a particular class of solutions, called the Jacobi elliptic functions, make them useful for the analytical solution of a wide range of nonlinear problems. Using this powerful method, we derive a set of exact solutions for the Shynaray-IIA equation, shedding light on its complex dynamics and behaviour. The proposed method is shown to be highly effective in obtaining exact solutions in terms of Jacobi elliptic functions, such as dark, bright, periodic, dark-bright, dark-periodic, bright periodic, singular, and other various types of solitons. Furthermore, a detailed analysis is conducted on the convergence and accuracy of the obtained solutions. The outcomes of this study extend the applicability of the Jacobi elliptic function approach to a novel class of non-linear models and provide valuable insights into the dynamics of Shynaray-IIA equation. This study advances the creation of efficient mathematical instruments for resolving intricate nonlinear phenomena across a range of scientific fields.

Key words: soliton theory, integrable partial differential equation, analytical approach, propagation of solitary waves

1. INTRODUCTION

The Shynaray-IIA is a coupled partial differential equation, a significant nonlinear partial differential equation (PDE), arises in numerous branches of physical and mathematical sciences, like as fluid mechanics, quantum physics and plasma physics. Its complex nonlinear nature presents a substantial challenge in finding exact analytical solutions, leading researchers to explore innovative and efficient methods for resolution such as tanh method $[1]$, extended auxiliary equation method $[2-4]$, variational method $[5]$, modified and extended simple equation method $[6-8]$, direct algebraic method $[9]$, generalized exponential rational function technique $[10]$, extended F-expansion scheme $[11, 12]$, $G/_{G'}$ – expansion algorithm $[13]$, sine-Gordon expansion method $[14]$, modified sub-equation method $[15]$, darboux method $[16]$, homogeneous balance $[17]$, and so on $[18-33]$. Among the abundance of mathematical tools available, the Jacobi elliptic function approach has emerged as a promising scheme for solving the non-linear partial differential equations (PDEs). This technique is particularly valuable in handling nonlinear equations with high nonlinearity, as it enables researchers to obtain exact solutions by transforming the original equation into a more manageable elliptic equation. In this research article, we focus on investigating the application of the Jacobi elliptic function approach to handle the Shynaray-IIA equation. The considered model is given as,

$$
\begin{aligned}
&iq_t + q_{xt} - i(vq)_x = 0,\\
&ir_t - r_{xt} - i(vr)_x = 0,\\
&v_x - \frac{n^2}{\alpha}(rq)_t = 0.
\end{aligned} \tag{1}
$$

We aimed to construct exact analytical solutions that shed light on the intricate dynamics described by the equation. The obtained solutions not only contribute to a deeper understanding of underlying physical processes but also offer a valuable standard for validating numerical and approximate method in solving this challenging PDE. The Jacobi elliptic function expansion method serves as a powerful mathematical tool to solve the Shynaray-IIA (S-IIA) equation, allowing us to gain deeper insight into the behavior of complex physical systems. The exact analytical solutions obtained through this research contribute to the existing body of knowledge, paving the way for further advancement in the study of nonlinear Partial differential equations (PDEs) and their implications in diverse scientific disciplines. Sachin et al. $[34-38]$ have examined the Konopelchenko–Dubrovsky (KD) equation, generalized complex coupled Schrödinger–Boussinesq equations, Sakovich equation, Zakharov–Kuznetsov–Benjamin–Bona–Mahony (ZK-BBM) equation and Vakhnenko-Parkes equation to develop the solitary wave solution and visualized their propagation by utilizing the distinct analytical techniques. Rani et al. [39] constructed exact analytical solutions for complex modified Kortewegde-Vries. Nonlaopon et al. [40] performed Painlevé analysis to form the exact soliton solutions.

The remainder of this article is presented in the following structure: Section 1, provides a brief overview of the Shynaray-IIA equation and its relevance in various scientific fields. Section II outlines the theoretical basis of considered method. In Section III, we present the step-by-step implementation of the method to obtain exact solutions for the Shynaray-IIA equation. In section IV, provide the analysis of graphs for direct study. Section V, discusses the conclusion and applicability of the proposed approach.

2. DESCRIPTION OF ANALYTICAL TECHNIQUE

An overview of the Jacobi elliptic function methodology is given in this section. We will use nonlinear partial differential equations, which typically have the following mathematical conclusion,

$$N(u,\frac{\partial u}{\partial t},\frac{\partial u}{\partial x},\frac{\partial^2 u}{\partial t^2},\frac{\partial^2 u}{\partial x^2},\dots)=0. \quad (2)$$

Utilizing the following wave transformation to convert Eq. (1) into an ordinary differential equation,

$$u=u(\xi),\xi=k(x-ct), \quad (3)$$

where the symbols for frequency and wave number, respectively, are c and k. Equation (1) has been successfully transformed into an ordinary differential equation (ODE) by the procedure described in Equation (2).

$$\frac{\partial(.)}{\partial t}=-ck\frac{d(.)}{d\xi},\ \frac{\partial(.)}{\partial x}=k\frac{d(.)}{d\xi}, \quad (4)$$

$$N(u',u'',u''',\dots.)=0. \quad (5)$$

Tab. 1. The chosen value of P, Q and R

	P	**Q**	**R**	**F**
1	m^2	$-(1+m^2)$	1	sn, cd
2	$-m^2$	$2m^2-1$	$1-m^2$	cn
3	-1	$2-m^2$	m^2-1	dn
4	1	$-(1+m^2)$	m^2	ns, dc
5	$1-m^2$	$2m^2-1$	$-m^2$	nc
6	m^2-1	$2-m^2$	-1	nd
7	$1-m^2$	$2-m^2$	1	sc
8	$-m^2(1-m^2)$	$2m^2-1$	1	sd
9	1	$2-m^2$	$1-m^2$	cs
10	1	$2m^2-1$	$-m^2(1-m^2)$	ds
11	$\frac{-1}{4}$	$\frac{m^2+1}{2}$	$\frac{-(1-m^2)^2}{4}$	$mcn \mp dn$
12	$\frac{1}{4}$	$\frac{-2m^2+1}{2}$	$\frac{1}{4}$	$ns \mp cs$
13	$\frac{1-m^2}{4}$	$\frac{m^2+1}{2}$	$\frac{1-m^2}{4}$	$nc \mp sc$
14	$\frac{1}{4}$	$\frac{m^2-2}{2}$	$\frac{m^4}{4}$	$ns \mp ds$
15	$\frac{m^2}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	$sn \mp icn, \frac{sn}{\sqrt{1-m^2sn}}$
16	$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$	$mcn \mp idn, \frac{sn}{1\mp cn}$
17	$\frac{m^2}{4}$	$\frac{m^2-2}{2}$	$\frac{1}{4}$	$\frac{sn}{1\mp dn}$
18	$\frac{m^2-1}{4}$	$\frac{m^2+1}{2}$	$\frac{m^2-1}{4}$	$\frac{dn}{1\mp msn}$
19	$\frac{1-m^2}{4}$	$\frac{m^2+1}{2}$	$\frac{-m^2+1}{4}$	$\frac{cn}{1\mp sn}$
20	$\frac{(1-m^2)^2}{4}$	$\frac{m^2+1}{2}$	$\frac{1}{4}$	$\frac{sn}{dn\mp cn}$
21	$\frac{m^4}{4}$	$\frac{m^2-2}{2}$	$\frac{1}{4}$	$\frac{cn}{\sqrt{1-m^2\mp dn}}$

In conjunction with utilizing this advanced directed technique, the underlying principle entails augmenting the likelihood of resolving an auxiliary ODE, namely first-order Jacobian problem with the three parameters. This method aims to produce a multitude of Jacobian elliptic solutions for the given issue. Visualizing the auxiliary equation is a feasible step in understanding this process.

$$(F')^2(\xi)=PF^4(\xi)+QF^2(\xi)+R. \quad (6)$$

Let $F'=\frac{dF}{d\xi}$, where $\xi=\xi(x,t)$, and the constants P, Q and R are involved. The solution for equation (5) is provided in Tab. 1. It is important to note that $i^2=-1$. Additionally, the Jacobi elliptic functions are denoted as $sn\xi=sn(\xi,m), cn\xi=cn(\xi,m)$, and $dn\xi=dn(\xi,m)$, where m lies in the range $0<m<1$ and represents the modulus.

Tab. 2. Analysis of Jacobi elliptic functions in the limit of $m\to 0\ and\ m\to 1$.

		$m\to 1$	$m\to 0$			$m\to 1$	$m\to 0$
1	snu	$tanhu$	$sinu$	7	dcu	1	$secu$
2	cnu	$sechu$	$cosu$	8	ncu	$coshu$	$secu$
3	dnu	$sechu$	1	9	scu	$sinhu$	$tanu$
4	cdu	1	$cosu$	10	nsu	$cothu$	$cscu$
5	sdu	$sinhu$	$sinu$	11	dsu	$cschu$	$cscu$
6	ndu	$coshu$	1	12	csu	$cschu$	$cotu$

The elliptic functions exhibit a distinctive double periodic, processing distinct properties as outline below:

$$sn^2\xi+cn^2\xi=1,$$

$$dn^2\xi+m^2sn^2\xi=1,$$

$$\frac{d}{d\xi}sn\xi=cn\xi dn\xi,$$

$$\frac{d}{d\xi}cn\xi=-sn\xi dn\xi,$$

$$\frac{d}{d\xi}dn\xi=-m^2sn\xi cn\xi.$$

With reference to Tab. 2, this reduction makes it possible to derive the solutions for the given problem using the trigonometric function and solitons. The Jacobi elliptic function expansion method can be used to describe the function as a finite series of Jacobi elliptic functions.

$$u(\xi) = \sum_{i=0}^{n} a_i F^i(\xi). \tag{7}$$

Here the function $F(\xi)$ represents solution to the non-linear ordinary equation denoted as Eq. 5. The constants n and a_i (where $i = 0, 1, 2, \ldots, n$) are parameters that have to be found. The determination of the integer n in Eq. 6 involves an analysis of the highest order linear term.

$$O\left(\frac{d^p u}{d\xi^p}\right) = n + p, \; p = 0,1,2,3, \ldots \ldots, \tag{8}$$

thus, the most significant nonlinear terms at the highest order are

$$O\left(u^q \frac{d^p u}{d\xi^p}\right) = (q+1)n + p, \; p = 0,1,2,3, \ldots \ldots,$$

$$q = 1,2,3, \ldots, \tag{9}$$

in Eq. 4.

Utilizing Eq. 6 and setting all coefficients of powers F to zero, we derive a set of nonlinear algebraic equations for the variablesa_i, (where $i = 0,1,2,3, \ldots.$). Employing Mathematica, we proceed to solve this system of algebraic equations and put all the values for P, Q, and R as per Eq. 5 in Tab. 1. This approach, integrating the information from Eq. 6 with the selected auxiliary equation, allows for the determination of exact solutions for Eq. 1.

3. THE CONSTRUCTION OF SOLITONS OF SHYNARAY-IIA EQUATION (S-IIAE)

The precise solutions to Shynaray-IIA Eq. 1 using the Jacobi elliptic function expansion approach are shown in this section,

$$iq_t + q_{xt} - i(vq)_x = 0,$$

$$ir_t - r_{xt} - i(vr)_x = 0,$$

$$v_x - \frac{n^2}{\alpha}(rq)_t = 0.$$

In case when $= \epsilon\bar{q}$ $(\epsilon = \pm 1)$, the S-IIAE takes the following form:

$$iq_t + q_{xt} - i(vq)_x = 0,$$

$$v_x - \frac{n^2 \epsilon}{\alpha}(|q|^2)_t = 0. \tag{10}$$

In the above equation m, n and ϵ are constants. By using the traveling wave transformation Eq. 11 is reduced into the following ODE:

$$q(x,t) = U(\eta)e^{i\xi(x,t)}, v(x,t) = G(\eta),$$

$$\xi(x,t) = -\delta x + \omega t + \theta, \eta = x - ct, \tag{11}$$

where $v, \theta, \omega, \delta$ characterize the frequency, the phase constant, the wave number and the velocity, respectively. The Eq. 27 is plugging into the first part of Eq. 26 and getting the real and imaginary parts,

$$cU''(\eta) + \omega(1-\delta)U(\eta) + \delta G(\eta)U(\eta) + i\big(\omega - c(1-\delta)\big)U'(\eta) - G(\eta)U'(\eta) - G'(\eta)U(\eta) = 0,$$

$$G'(\eta) + \frac{2c\epsilon n^2}{\alpha}U(\eta)U'(\eta) = 0. \tag{12}$$

The second Eq. 28 is integrated, and we get

$$G(\eta) = -\frac{c\epsilon n^2}{\alpha}U^2(\eta). \tag{13}$$

Substitute the Eq. 13 into the first part of 12 and separating the real and imaginary parts as

$$cU''(\eta) + \omega(1-\delta)U(\eta) - \frac{\delta c\epsilon n^2}{\alpha}U^3(\eta) = 0. \tag{14}$$

And we have the imaginary part as,

$$\big(\omega - c(1-\delta)\big)U'(\eta) + \frac{3c\epsilon n^2}{\alpha}U''(\eta)U'(\eta) = 0. \tag{15}$$

By using the homogeneous balancing procedure, we obtained $n = 1$, the determine value of n is substituted in Eg. 7 we obtained the simple form of the solution as:

$$U(\eta) = a_0 + a_1 F(\eta), \tag{16}$$

$$U^3(\eta) = a_0^3 + a_1^3 F^3(\eta) + 3a_0 a_1^2 F^2(\eta) + 3a_0^2 a_1 F \tag{17}$$

and

$$U''(\eta) = a_1\big(2PF^3(\eta) + QF(\eta)\big). \tag{18}$$

Substitute Eq. 16-18 into Eq. 15, we get,

$$ca_1\big(2PF^3(\eta) + QF(\eta)\big) + \omega(1-\delta)\big(a_0 + a_1F(\eta)\big) - \frac{\delta c\epsilon n^2}{\alpha}\big(a_0^3 + a_1^3F^3(\eta) + 3a_0a_1^2F^2(\eta) + 3a_0^2a_1F(\eta)\big) = 0. \tag{19}$$

By collecting the various coefficients of $F^i(\eta)$, we get the system of equations,

$$U^0: \quad (\omega(1-\delta) - \frac{\delta c\epsilon n^2}{\alpha}a_0^2)\, a_0 = 0, \tag{20}$$

$$U^1: (cQ + \omega(1-\delta) - 3a_0^2\frac{\delta c\epsilon n^2}{\alpha})a_1 = 0, \tag{21}$$

$$U^2: -3a_0a_1^3\frac{\delta c\epsilon n^2}{\alpha} = 0, \tag{22}$$

$$U^3 = (2Pc - a_1^2\frac{\delta c\epsilon n^2}{\alpha})a_1 = 0. \tag{23}$$

Upon solving the aforementioned system by using Maple software, we obtain the coefficients pertaining to the series 16,

$$a_0 = 0, \; a_1 = \pm\frac{\sqrt{\frac{2\alpha P}{\delta\epsilon}}}{n}. \tag{24}$$

The obtained solution is of the form,

$$U = \pm\frac{\sqrt{\frac{2\alpha P}{\delta\epsilon}}}{n}\,F(\eta). \tag{25}$$

When the values $P = m^2$, $Q = -(1+m^2)$, and $R = 1$ are chosen, table 1 provides the corresponding values of $F = sn$. Therefore, the periodic solution of Equation 1 can be represented as,

$$q_{1,1} = \pm\frac{\sqrt{\frac{2\alpha m^2}{\delta\epsilon}}}{n}\; sn(x-ct), \tag{26}$$

$$v_{1,1} = -\frac{2cm^2}{\delta}sn^2(x-ct). \tag{27}$$

Supposing $m \to 1$, hence, by referring to table 2, one may derive the solitary wave solution of Eq. 1.

$$q_{1,2} = \pm\frac{\sqrt{\frac{2\alpha}{\delta\epsilon}}}{n}\; tanh(x-ct), \tag{28}$$

$$v_{1,2}=-\frac{2c}{\delta}tanh^2(x-ct). \quad (29)$$

Choosing $P=-m^2, Q=2m^2-1, R=1-m^2$, based on the data supplied in Table 1, it can be inferred that the variable F can be mathematically expressed as $F=cn$. Consequently, the periodic solution of Equation (1) can be derived as follows:

$$q_{1,3}=\pm\frac{\sqrt{\frac{-2\alpha m^2}{\delta\epsilon}}}{n}\,cn(x-ct), \quad (30)$$

$$v_{1,3}=\frac{2cm^2}{\delta}cn^2(x-ct). \quad (31)$$

Considering $m\to 1$ the solitary wave solution of Eq. 1 can be expressed as per the information provided in Tab. 2.

$$q_{1,4}=\pm\frac{\sqrt{\frac{-2\alpha}{\delta\epsilon}}}{n}\,sech(x-ct), \quad (32)$$

$$v_{1,4}=\frac{2c}{\delta}sech^2(x-ct). \quad (33)$$

Setting $P=-1,\ \ Q=2-m^2,\ \ R=m^2-1$, based on the data shown in Tab. 1, it can be inferred that the periodic solution of Eq. 1 can be mathematically represented as follows:

$$q_{1,5}=\pm\frac{\sqrt{\frac{-2\alpha}{\delta\epsilon}}}{n}\,dn(x-ct), \quad (34)$$

$$v_{1,5}=\frac{2c}{\delta}dn^2(x-ct). \quad (35)$$

In the context of $m\to 1$ from Tab. 2, the similarity between the solution shown and the solution derived in Eq. 27 is clearly demonstrated.

While $P=1, Q=-(1+m^2), R=m^2, F=ns$, according to the data presented in Tab. 1, the answer to Eq. 1 can be represented as follows:

$$q_{1,6}=\pm\frac{\sqrt{\frac{2\alpha}{\delta\epsilon}}}{n}\,ns(x-ct), \quad (36)$$

$$v_{1,6}=-\frac{2c}{\delta}ns^2(x-ct). \quad (37)$$

Additionally, when $m\to 1$ the solitary wave solution of Eq. 1 is presented in Tab. 2.

$$q_{1,7}=\pm\frac{\sqrt{\frac{2\alpha}{\delta\epsilon}}}{n}\,coth(x-ct), \quad (38)$$

$$v_{1,7}=-\frac{2c}{\delta}coth^2(x-ct). \quad (39)$$

Using Tab. 2, the periodic solution of Eq. 1 can be stated as follows if m⟶0:

$$q_{1,8}=\pm\frac{\sqrt{\frac{2\alpha}{\delta\epsilon}}}{n}\,csc(x-ct), \quad (40)$$

$$v_{1,8}=-\frac{2c}{\delta}csc^2(x-ct). \quad (41)$$

Supposing $P=1, Q=-(1+m^2), R=m^2$.
Thus, $F=dc$,

$$q_{1,8}=\pm\frac{\sqrt{\frac{2\alpha}{\delta\epsilon}}}{n}\,dc(x-ct), \quad (42)$$

$$v_{1,8}=-\frac{2c}{\delta}dc^2(x-ct). \quad (43)$$

Using Tab. 2, the periodic solution of Eq. 1 can be stated as follows if m⟶0:

$$q_{1,10}=\pm\frac{\sqrt{\frac{2\alpha}{\delta\epsilon}}}{n}\,sec(x-ct), \quad (44)$$

$$v_{1,10}=-\frac{2c}{\delta}scc^2(x-ct). \quad (45)$$

When $P=1-m^2, Q=2m^2-1, R=-m^2. Thus, F=nc$ and the solution of periodic nature of Eq. 1 as:

$$q_{1,11}=\pm\frac{\sqrt{\frac{2\alpha(1-m^2)}{\delta\epsilon}}}{n}\,nc(x-ct), \quad (46)$$

$$v_{1,11}=-\frac{2c(1-m)^2}{\delta}nc^2(x-ct). \quad (47)$$

As $m\to 0$ from Tab. 2, it is shown that the solution found as that of 33.

Also regarding $P=1-m^2,\ \ Q=2-m^2, R=1.$
Thus, $F=sc$:

$$q_{1,12}=\pm\frac{\sqrt{\frac{2\alpha(1-m^2)}{\delta\epsilon}}}{n}\,sc(x-ct), \quad (48)$$

$$v_{1,12}=-\frac{2c(1-m)^2}{\delta}sc^2(x-ct). \quad (49)$$

Furthermore, we find the periodic solution of Eq. 1 as follows for m⟶0, as shown in Tab. 2:

$$q_{1,13}=\pm\frac{\sqrt{\frac{2\alpha}{\delta\epsilon}}}{n}\,tan(x-ct), \quad (50)$$

$$v_{1,13}=-\frac{2c}{\delta}tan^2(x-ct). \quad (51)$$

Considering $P=1,\ \ Q=2-m^2, R=1-m^2$ and $F=cs$, thus:

$$q_{1,14}=\pm\frac{\sqrt{\frac{2\alpha}{\delta\epsilon}}}{n}\,cs(x-ct), \quad (52)$$

$$v_{1,14}=-\frac{2c}{\delta}cs^2(x-ct). \quad (53)$$

The solitary wave solution Eq. (1) is given as follows as m⟶1, per Tab. 2:

$$q_{1,15}=\pm\frac{\sqrt{\frac{2\alpha}{\delta\epsilon}}}{n}\,csch(x-ct), \quad (54)$$

$$v_{1,15}=-\frac{2c}{\delta}csch^2(x-ct). \quad (55)$$

The solitary wave solution Eq. 1 is given as follows as m⟶0, per Tab. 2,

$$q_{1,16}=\pm\frac{\sqrt{\frac{2\alpha}{\delta\epsilon}}}{n}\,cot(x-ct), \quad (56)$$

$$v_{1,16}=-\frac{2c}{\delta}cot^2(x-ct). \quad (57)$$

Also assigning $P=1$, $Q=2m^2-1,\ \ R=m^4-m^2$ and $F=ds$. Thus,

$$q_{1,17}=\pm\frac{\sqrt{\frac{2\alpha}{\delta\epsilon}}}{n}\,ds(x-ct), \quad (58)$$

$$v_{1,17}=-\frac{2c}{\delta}dc^2(x-ct). \quad (59)$$

In this family, the soliton solution is the similar to Eq. 30. If the limit of $m\to 0$, the solution can be articulated as per Eq. 38 with reference to Tab. 2.

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Assuming P, Q, R as $P=\frac{-1}{4}, Q=\frac{m^2+1}{2}, R=\frac{-(1-m^2)}{4}$, according to Tab. 1, F formulated as $F=mcn\mp dn$, the solution is determined as,

$$q_{1,18}=\pm\frac{\sqrt{\frac{-\alpha}{2\delta\epsilon}}}{n}\, mcn(x-ct)\mp dn(x-ct), \quad (60)$$

$$v_{1,18}=\frac{c}{2\delta}(mcn(x-ct)\mp dn(x-ct))^2. \quad (61)$$

Additionally, when $m\to 1$, the obtained solution is similar the solution found in Eq. (25).

If we select P, Q, R as $P=\frac{1}{4}$, $Q=\frac{-2m^2+1}{2}, R=\frac{1}{4}$, and evaluate F from table 1 where $= ns\mp cs$, thus solution of Eq. (1) can be indicated as,

$$q_{1,19}=\pm\frac{\sqrt{\frac{\alpha}{2\delta\epsilon}}}{n}\big(ns(x-ct)\mp cs(x-ct)\big), \quad (62)$$

$$v_{1,19}=-\frac{c}{2\delta}(ns(x-ct)\mp cs(x-ct))^2. \quad (63)$$

The solitary wave solution for $m\to 1$ in Eq. 1 is identified as,

$$q_{1,20}=\pm\frac{\sqrt{\frac{\alpha}{2\delta\epsilon}}}{n}\, coth(x-ct)\mp csch(x-ct), \quad (64)$$

$$v_{1,20}=-\frac{c}{2\delta}(\coth(x-ct)\mp \operatorname{csch}(x-ct))^2. \quad (65)$$

Additionally, in the case where $m\to 0$, based on Tab. 2, obtaining a periodic solution is evident.

$$q_{1,21}=\pm\frac{\sqrt{\frac{\alpha}{2\delta\epsilon}}}{n}\, csc(x-ct)\mp cot(x-ct), \quad (66)$$

$$v_{1,21}=-\frac{c}{2\delta}(\csc(x-ct)\mp \cot(x-ct))^2. \quad (67)$$

If $P=\frac{1-m^2}{4}, Q=\frac{m^2+1}{2}, R=\frac{1-m^2}{4}$ and $F=nc\mp sc$, the solution of Eq. (1) can be found as,

$$q_{1,22}=\pm\frac{\sqrt{\frac{\alpha(1-m^2)}{2\delta\epsilon}}}{n}\,(nc(x-ct)\mp sc(x-ct)), \quad (68)$$

$$v_{1,22}=\frac{-c(1+m^2)}{2\delta}\,(nc(x-ct)\mp sc(x-ct))^2. \quad (69)$$

The solitary wave solution for $m\to 0$ in Eq. 1 is identified as,

$$q_{1,23}=\pm\frac{\sqrt{\frac{\alpha}{2\delta\epsilon}}}{n}\,(sec(x-ct)\mp tan(x-ct)), \quad (70)$$

$$v_{1,23}=-\frac{c}{2\delta}(\sec(x-ct)\mp\tan(x-ct))^2. \quad (71)$$

Setting $P=\frac{m^2}{4}, Q=\frac{m^2-2}{2}, R=\frac{m^2}{4}$, as per Table 1, $F=sn\mp icn$, due to this setting the solution of Eq. 1 can be found as:

$$q_{1,24}=\pm\frac{\sqrt{\frac{\alpha m^2}{2\delta\epsilon}}}{n}\,(sn(x-ct)\mp icn(x-ct)), \quad (72)$$

$$v_{1,24}=-\frac{cm^2}{2\delta}(sn(x-ct)\mp icn(x-ct))^2. \quad (73)$$

The solitary wave solution for $m\to 1$ in Eq. (1) is identified as,

$$q_{1,25}=\pm\frac{\sqrt{\frac{\alpha}{2\delta\epsilon}}}{n}\,\big(tanh(x-ct)\mp isech(x-ct)\big), \quad (74)$$

$$v_{1,25}=\frac{-c}{2\delta}\,(\tanh(x-ct)\mp isech(x-ct))^2. \quad (75)$$

Regarding $P=\frac{1}{4}, Q=\frac{-2m^2+1}{2}, R=\frac{1}{4}$, and $F=msn\mp idn$ from the Table 1, thus, the solution of Eq. 1 can be expressed as,

$$q_{1,26}=\pm\frac{\sqrt{\frac{\alpha}{2\delta\epsilon}}}{n}\,(msn(x-ct)\mp idn(x-ct)), \quad (76)$$

$$v_{1,26}=\frac{-c}{2\delta}\,(msn(x-ct)\mp idn(x-ct))^2. \quad (77)$$

For $m\to 1$, the solution obtained as that of 48.

Considering

$$P=\frac{1}{4}, Q=\frac{1-2m^2}{2}, R=\frac{1}{4},$$

and

$$F=\frac{sn}{1\mp cn},$$

from Tab. 1, thus, the solution of Eq. 1 can be found as,

$$q_{1,27}=\pm\frac{\sqrt{\frac{\alpha}{2\delta\epsilon}}}{n}\,\frac{sn(x-ct)}{1\mp cn(x-ct)}, \quad (78)$$

$$v_{1,27}=\frac{-c}{2\delta}\,\Big(\frac{sn(x-ct)}{1\mp cn(x-ct)}\Big)^2. \quad (79)$$

If we take a look at Tab. 2, we can determine the solitary wave solution of Eq. 1 for $m\to 1$,

$$q_{1,28}=\pm\frac{\sqrt{\frac{\alpha}{2\delta\epsilon}}}{n}\,\frac{tanh(x-ct)}{1\mp sech(x-ct)}, \quad (80)$$

$$v_{1,28}=\frac{-c}{2\delta}\,\Big(\frac{\tanh(x-ct)}{1\mp \operatorname{sech}(x-ct)}\Big)^2. \quad (81)$$

If we take a look at Tab. 2, we can determine the solitary wave solution of Eq. 1 for $m\to 0$,

$$q_{1,29}=\pm\frac{\sqrt{\frac{\alpha}{2\delta\epsilon}}}{n}\,\frac{sin(x-ct)}{1\mp cos(x-ct)}. \quad (82)$$

$$v_{1,29}=\frac{-c}{2\delta}\,\Big(\frac{\sin(x-ct)}{1\mp\cos(x-ct)}\Big)^2. \quad (83)$$

Supposing $P=\frac{m^2}{4}, Q=\frac{m^2-2}{2}, R=\frac{1}{4}$ it can be concluded from Tab. 1 $F=\frac{sn}{1\mp dn}$, so the solution of Eq. 1 can be found as,

$$q_{1,30}=\pm\frac{\sqrt{\frac{\alpha m^2}{2\delta\epsilon}}}{n}\,\frac{sn(x-ct)}{1\mp dn(x-ct)}, \quad (84)$$

$$v_{1,30}=-\frac{cm^2}{2\delta}\Big(\frac{sn(x-ct)}{1\mp dn(x-ct)}\Big)^2. \quad (85)$$

When $m\to 1$, the solution is determined by the solution in equation 52.

From Tab. 1, allocating $P=\frac{1-m^2}{4}, Q=\frac{m^2+1}{2}, R=\frac{1-m^2}{4}$ and

$$F=\frac{cn}{1\mp sn},$$

thus,

$$q_{1,31}=\pm\frac{\sqrt{\frac{\alpha(1-m^2)}{2\delta\epsilon}}}{n}\,\frac{cn(x-ct)}{1\mp sn(x-ct)}, \quad (86)$$

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$$v_{1,31}=\frac{-c(1+m^2)}{2\delta}\left(\frac{cn(x-ct)}{1\mp sn(x-ct)}\right)^2. \quad (87)$$

If we take a look at Tab. 2, we can determine the solitary wave solution of Eq. 1 for $m\to 0$,

$$q_{1,32}=\pm\frac{\sqrt{\frac{\alpha}{2\delta\epsilon}}}{n}\frac{cos(x-ct)}{1\mp sin(x-ct)}, \quad (88)$$

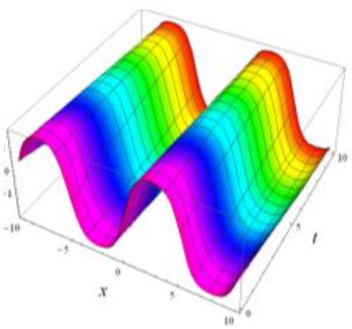
$$v_{1,32}=\frac{-c}{2\delta}\left(\frac{\cos(x-ct)}{1\mp\sin(x-ct)}\right)^2. \quad (89)$$

Choosing $P=\frac{(1-m^2)^2}{4}, Q=\frac{m^2+1}{2}, R=\frac{1}{4}$ and $F=\frac{sn}{dn\mp cn}$, so that the solution of Eq. 1 can be obtained as,

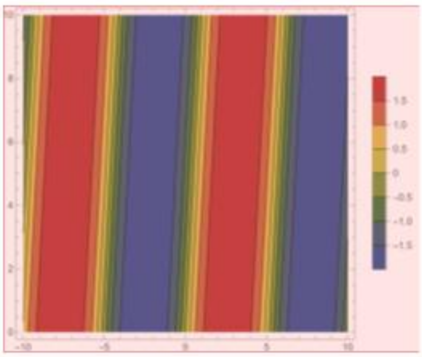
$$q_{1,33}=\pm\frac{\sqrt{\frac{\alpha(1-m^2)^2}{2\delta\epsilon}}}{n}\frac{sn(x-ct)}{dn(x-ct)\mp cn(x-ct)}, \quad (90)$$

$$v_{1,33}=\frac{-c(1+m^2)^2}{2\delta}\left(\frac{sn(x-ct)}{dn(x-ct)\mp cn(x-ct)}\right)^2. \quad (91)$$

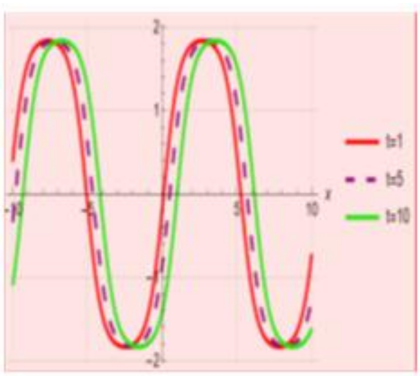
For $m\to 0$, the solution is obtained as that of 51.



a) 3-D visualization

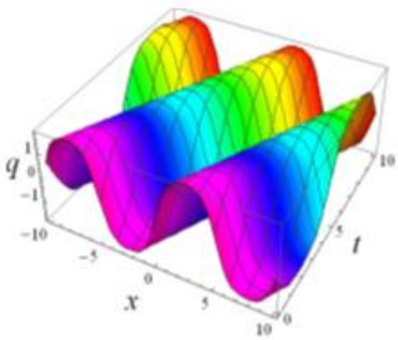


b) contour visualization

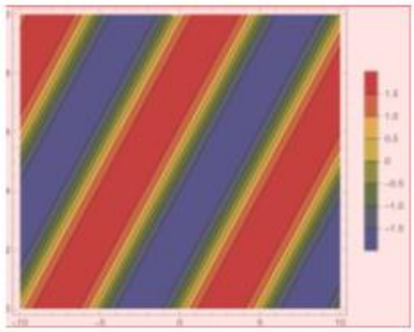


c) 2-D visualization

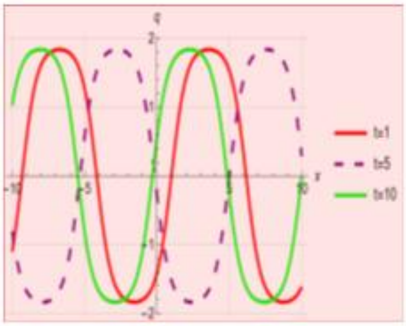
Fig. 1. 3-D, contour visualization and 2-D propagation of $q_{1,1}$ for specific values of the parameters are $\epsilon=1.2$, $\alpha=1.3$, $\delta=0.5$, $m=0.9$, $c=0.1$



a) 3-D visualization

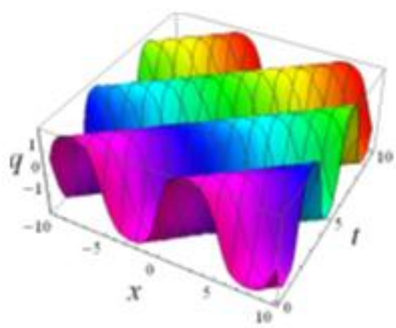


b) contour visualization



c) 2-D visualization

Fig. 2. 3-D, contour visualization and 2-D propagation of $q_{1,1}$ for specific values of the parameters are $\epsilon=1.2$, $\alpha=1.3$, $\delta=0.5$, $m=0.9$, $c=01$



a) 3-D visualization

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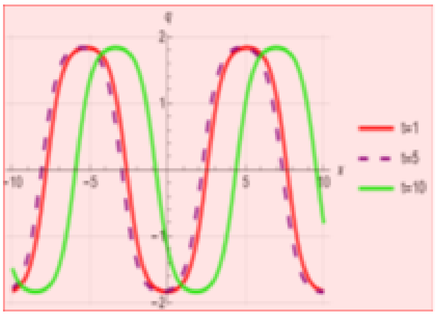
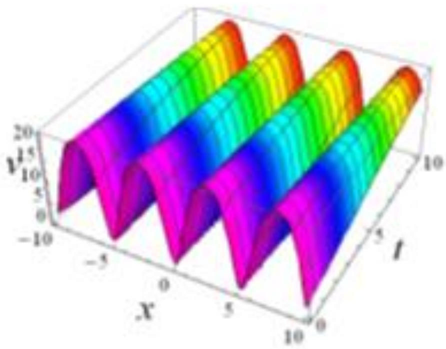
b) contour visualization

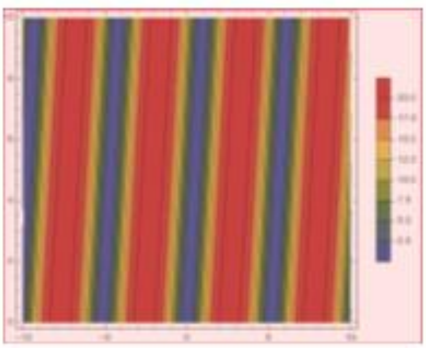


c) 2-D visualization

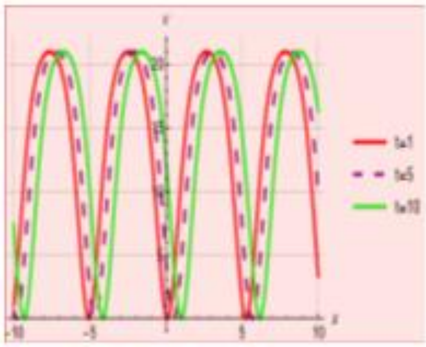
Fig. 3. 3-D, contour visualization and 2-D propagation of $q_{1,1}$ for specific values of the parameters are $\epsilon = 1.2, \alpha = 1.3, \ \delta = 0.5, m = 0.9, c = 2.5$



a) 3-D visualization

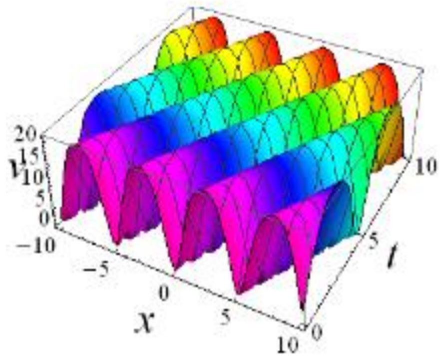


b) contour visualization

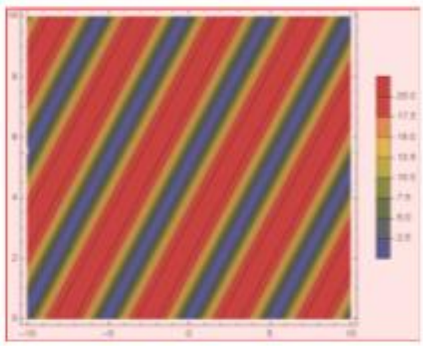


c) 2-D visualization

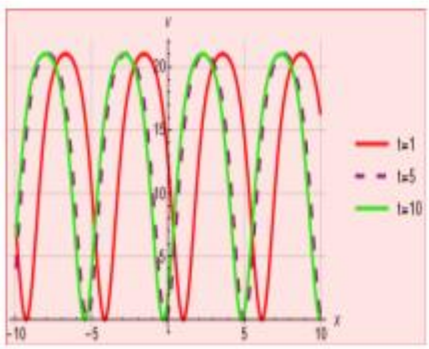
Fig. 4. 3-D, contour visualization and 2-D propagation of $v_{1,1}$ for specific values of the parameters are $\epsilon = 1.2, \alpha = 1.3, \ \delta = 0.5, n = 1.5, c = -1.5, m = 0.9, c = 0.1.$



a) 3-D visualization

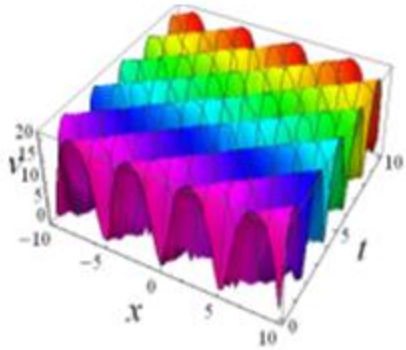


b) contour visualization

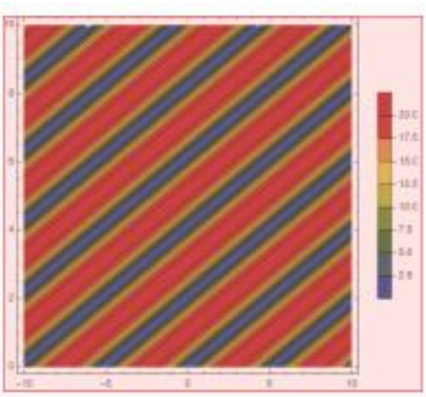


c) 2-D visualization

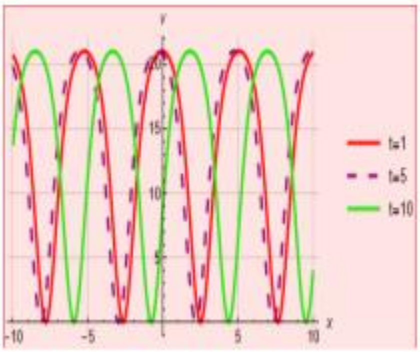
Fig. 5. 3-D, contour visualization and 2-D propagation of $v_{1,1}$ for specific values of the parameters are $\epsilon = 1.2, \alpha = 1.3, \ \delta = 0.5, n = 1.5, c = -1.5, m = 0.9, c = 01$



a) 3-D visualization

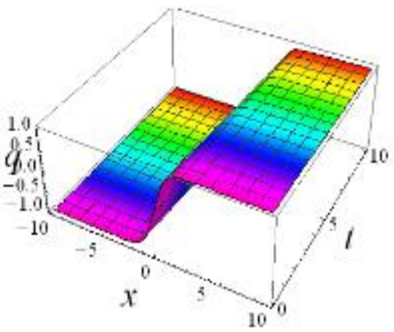


b) contour visualization

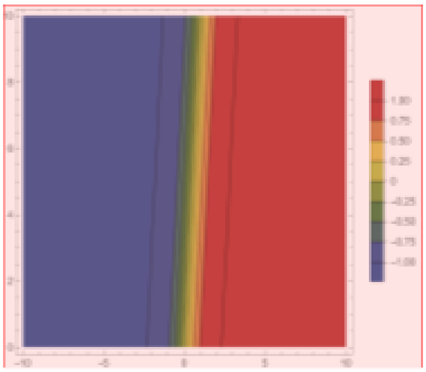


c) 2-D visualization

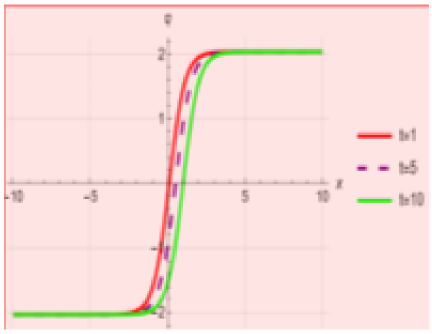
Fig. 6. 3-D, contour visualization and 2-D propagation of $v_{1,1}$ for specific values of the parameters are $\epsilon = 1.2, \alpha = 1.3, \ \delta = 0.5, n = 1.5, c = -1.5, m = 0.9, c = 2.5$



a) 3-D visualization

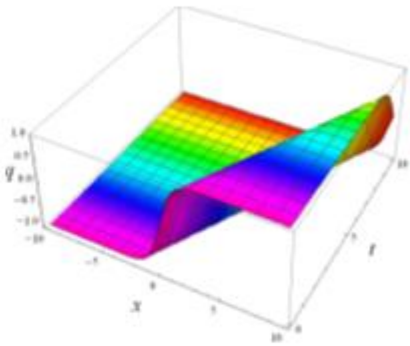


b) contour visualization

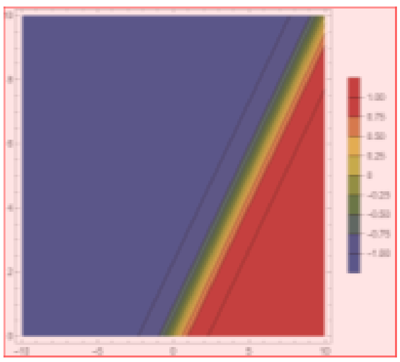


c) 2-D visualization

Fig. 7. 3-D, contour visualization and 2-D propagation of $q_{1,2}$ for specific values of the parameters are $\epsilon = 1.2, \alpha = 1.3, \ \delta = 0.5, m = 0.5, c = 0.1$



a) 3-D visualization



b) contour visualization

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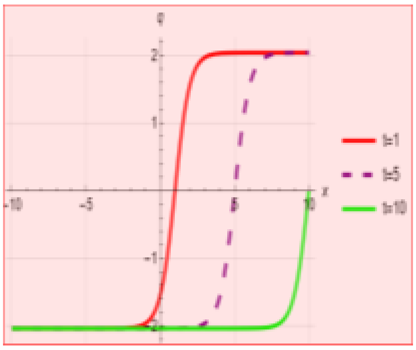
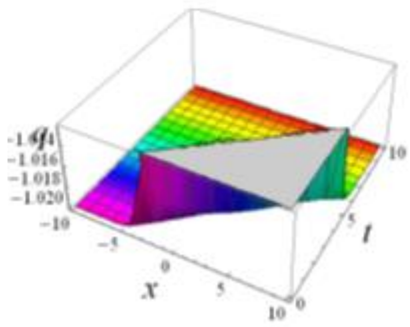
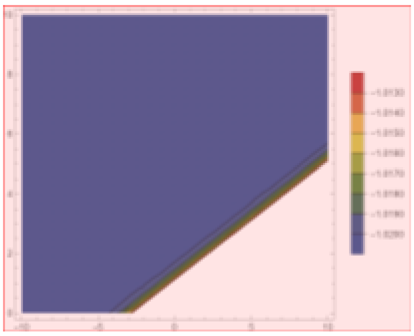


c) 2-D visualization

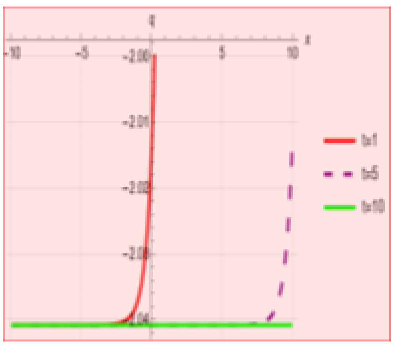
Fig. 8. 3-D, contour visualization and 2-D propagation of $q_{1,2}$ for specific values of the parameters are $\epsilon = 1.2, \alpha = 1.3, \delta = 0.5, m = 0.5, c = 01$



a) 3-D visualization

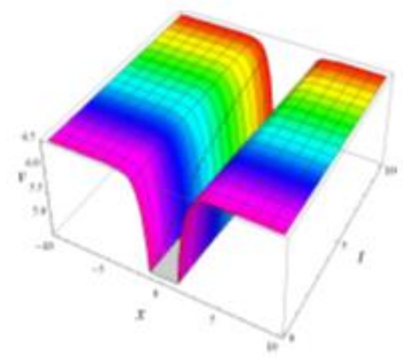


b) contour visualization

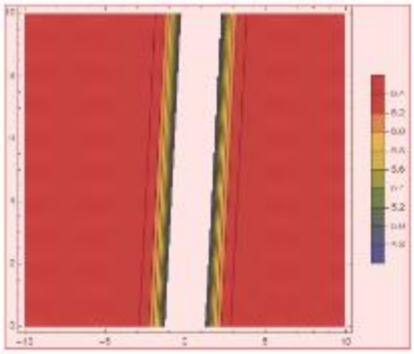


c) 2-D visualization

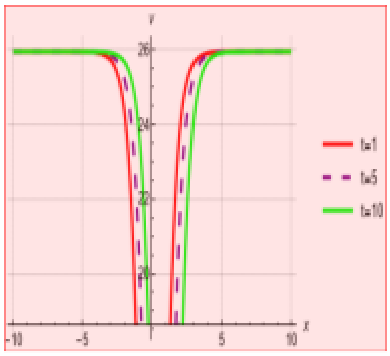
Fig. 9. 3-D, contour visualization and 2-D propagation of $q_{1,2}$ for specific values of the parameters are $\epsilon = 1.2, \alpha = 1.3, \delta = 0.5, m = 0.5, c = 2.5$



a) 3-D visualization

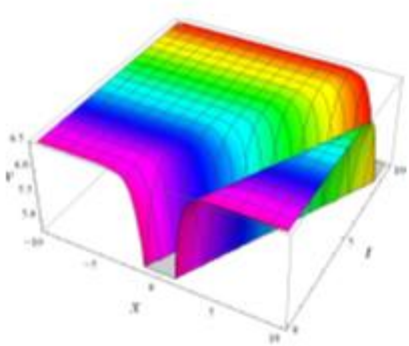


b) contour visualization



c) 2-D visualization

Fig. 10. 3-D, contour visualization and 2-D propagation of $v_{1,2}$ for specific values of the parameters are $\epsilon = 1.2, \alpha = 1.3, \delta = 0.5, n = 1.5, c = -1.5, m = 0.5, c = 0.1$



a) 3-D visualization

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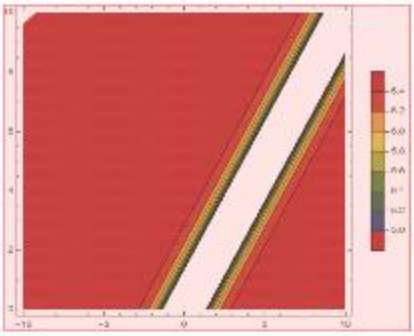
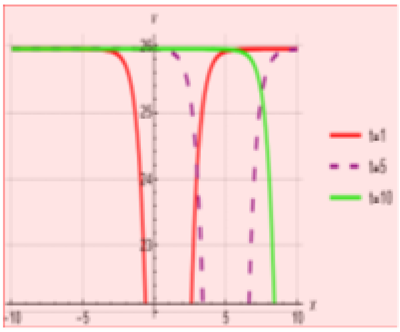
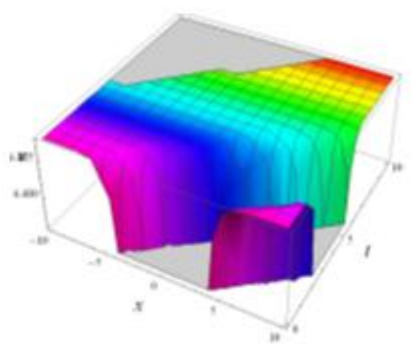


b) contour visualization

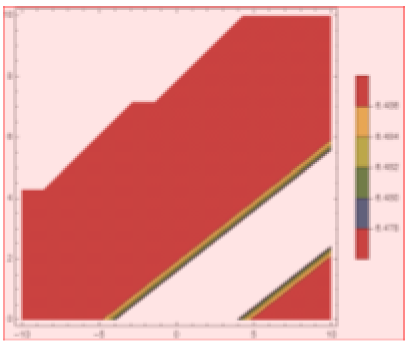


c) 2-D visualization

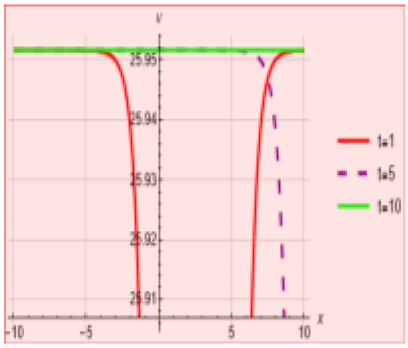
Fig. 11. 3-D, contour visualization and 2-D propagation of $v_{1,2}$ for specific values of the parameters are $\epsilon = 1.2, \alpha = 1.3, \delta = 0.5, n = 1.5, c = -1.5, m = 0.5, c = 01$



a) 3-D visualization



b) contour visualization



c) 2-D visualization

Fig. 12. 3-D, contour visualization and 2-D propagation of $v_{1,2}$ for specific values of the parameters are $\epsilon = 1.2, \alpha = 1.3, \delta = 0.5, n = 1.5, c = -1.5, m = 0.5, c = 2.5$

4. PHYSICAL EXPLANATIONS

This section offers physical explanation of Figure [1-12] and selection of wave solutions that have been obtained by applying the Jacobi elliptic function expansion method to the S-IIAE equation. In order to create visual representations of different soliton wave patterns, we have carefully selected and used certain parameter values. These patterns are illustrated in the accompanying figures. For every scenario, we have produced surface and contour visualization plots in two and three dimensions. These visual aids are important because they can verify that the theoretical conclusions, we came to earlier are accurate. It's important to keep in mind that these graphs and figures were produced using Mathematica. Consequently, one can notice that, the above-mentioned graphics are presenting the dark-bright, periodic, composite and bright soliton behavior respectively, under the influence of variation of wave number. On the other hand, the influence of wave is also discussed and noticed that, researchers and physicists can acquire their required results by controlling the propagation of soliton with wave number.

5. CONCLUSION

In conclusion, this research article explored the application of the Jacobi elliptic function expansion method for the Shynaray-IIA Equation (S-IIAE). The partial differential model is transformed into ordinary differential equation by employing the next travelling wave transformation according to considered analytical technique. Numerous properties of a particular class of solutions, called the Jacobi elliptic functions, make them useful for the analytical solution of a wide range of nonlinear problems. Using this powerful method, we derive a set of exact solutions for the Shynaray-IIA (S-IIA) equation, shedding light on its complex dynamics and behavior. The proposed method is shown to be highly effective in obtaining exact solutions in terms of Jacobi elliptic functions, such as dark, bright, periodic, dark-bright, dark-periodic, bright periodic, singular, and other various types of solitons. Additionally, a thorough examination of the accuracy and convergence of the obtained solutions is carried out. Overall, this research enriches the theoretical framework for the S-IIAE and presents a valuable tool for researchers and practitioners working in the field of nonlinear differential equations and mathematical physics.

REFERENCES

6. Wazwaz AM. Abundant solitons solutions for several forms of the fifth-order KdV equation by using the tanh method.Applied Mathematics and Computation. 2006;182(1):283-300.
7. Seadawy AR. Travelling-wave solutions of a weakly nonlinear two-dimensional higher-order Kadomtsev-Petviashvili dynamical equation for dispersive shallow-water waves. Eur. Phys. J. Plus. 2017;132:29. https://doi.org/10.1140/epjp/i2017-11313-4.
8. Akram U, Seadawy AR, Rizvi STR, Younis M, Althobaiti S, Sayed S. Traveling wave solutions for the fractional Wazwaz–Benjamin–Bona–Mahony model in arising shallow water waves. Results in Physics. 2021;20:103725. https://doi.org/10.1016/j.rinp.2020.103725
9. Rizvi STR, Kashif A, Marwa A. Optical solitons for Biswas–Milovic equation by new extended auxiliary equation method. Optik. 2020;204:164181. https://doi.org/10.1016/j.ijleo.2020.164181
10. Seadawy AR. New exact solutions for the KdV equation with higher order nonlinearity by using the variational method. Computers& Mathematics with Applications. 2011; 62(10):2011;3741-3755. https://doi.org/10.1016/j.camwa.2011.09.023
11. Asghar A, Seadawy AR, Dianchen L. Soliton solutions of the nonlinear Schrödinger equation with the dual power law nonlinearity and resonant nonlinear Schrödinger equation and their modulation instability analysis. Optik. 2017; 145;79-88. https://doi.org/10.1016/j.ijleo.2017.07.016
12. Arshad M, Seadawy AR, Dianchen L. Exact bright–dark solitary wave solutions of the higher-order cubic–quintic nonlinear Schrödinger equation and its stability. Optik. 2017;138;40-49. https://doi.org/10.1016/j.ijleo.2017.03.005
13. Arnous AH, Seadawy AR, Alqahtani RT, Biswas A. Optical solitons with complex Ginzburg–Landau equation by modified simple equation method. Optik. 2017;144:475-480. https://doi.org/10.1016/j.ijleo.2017.07.013
14. Seadawy AR, El-Rashidy K. Traveling wave solutions for some coupled nonlinear evolution equations,Mathematical and Computer Modelling. 2013;57(5-6):1371-1379. https://doi.org/10.1016/j.mcm.2012.11.026
15. Younas U, Younis M, Seadawy AR, Rizvi STR, Althobaiti S, Sayed S. Diverse exact solutions for modified nonlinear Schrödinger equation with conformable fractional derivative. Results in Physics. 2021,20;103766. https://doi.org/10.1016/j.rinp.2020.103766
16. Bhrawy AH, Abdelkawy MA, Kumar S, Biswas A. Solitons and other solutions to Kadomtsev-Petviashvili equation of B-type. Rom. J. Phys. 2013; 58(7-8):729-748.
17. Ebadi G, Fard NY, Bhrawy AH, Kumar S, Triki H, Yildirim A, Biswas A. Solitons and other solutions to the (3+ 1)-dimensional extended Kadomtsev-Petviashvili equation with power law nonlinearity. Rom. Rep. Phys. 2013; 65(1):27-62.
18. Iqbal MA, Wang Y, Miah MM, Osman MS. Study on date–Jimbo–Kashiwara–Miwa equation with conformable derivative dependent on time parameter to find the exact dynamic wave solutions. Fractal and Fractional. 2021; 6(1):4.
19. Ali KK, Wazwaz AM, Osman MS. Optical soliton solutions to the generalized nonautonomous nonlinear Schrödinger equations in optical fibers via the sine-Gordon expansion method. Optik. 2020; 208:164132.
20. Akinyemi L, Houwe A, Abbagari S, Wazwaz AM, Alshehri HM, Osman MS. Effects of the higher-order dispersion on solitary waves and modulation instability in a monomode fiber. Optik. 2023; 288: 171202. https://doi.org/10.1016/j.ijleo.2023.171202
21. Mani Rajan MS, Saravana Veni S, Wazwaz AM. Self-steepening nature and nonlinearity management of optical solitons with the influence of generalized external potentials. Opt Quant Electron. 2023; 55:703. https://doi.org/10.1007/s11082-023-04912-8
22. Jafari H, Tajadodi H, Baleanu D. Application of a homogeneous balance method to exact solutions of nonlinear fractional evolution equations. Journal of Computational and Nonlinear Dynamics. 2014;9(2): 021019.
23. Kumar S, Malik S, Rezazadeh H, Akinyemi L. The integrable Boussinesq equation and it's breather, lump and soliton solutions. Nonlinear Dynamics; 2022;1-14.
24. Kumar S, Kumar A., Samet B, Gómez-Aguilar JF, Osman MS. A chaos study of tumor and effector cells in fractional tumor-immune model for cancer treatment. Chaos, Solitons & Fractals 2020; 141: 110321.
25. Khan MI, Asghar S, Sabi'u J. Jacobi elliptic function expansion method for the improved modified kortwedge-de vries equation. Optical and Quantum Electronics. 2022; 54: 734.
26. Osman MS. One-soliton shaping and inelastic collision between double solitons in the fifth-order variable-coefficient Sawada–Kotera equation. Nonlinear Dynamics. 2019; 96(2):1491-1496.
27. Osman MS, Rezazadeh H., Eslami M. Traveling wave solutions for (3+1) dimensional conformable fractional Zakharov-Kuznetsov equation with power law nonlinearity. Nonlinear Engineering. 2016; 8(1):559-567.
28. Alquran M, Jarrah A. Jacobi elliptic function solutions for a two-mode KdV equation. J King Saud Univ Sci. 2019;31:485–9. https://doi.org/10.1016/j.jksus.2017.06.010
29. Jaradat HM, Syam M, Alquran M. A two-mode coupled Korteweg-de Vries: multiple-soliton solutions and other exact solutions. Nonlinear Dyn. 2017;90:371–7. https://doi.org/10.1007/s11071-017-3668-x
30. Alquran M, Jaradat HM, Syam MI. A modified approach for a reliable study of new nonlinear equation: two-mode Korteweg-de Vries-Burgers equation. Nonlinear Dyn. 2018;91:1619–26. https://doi.org/10.1007/s11071-017-3968-1
31. Jaradat HM, Awawdeh F, Al-Shara S, Alquran M, Momani S. Controllable dynamical behaviors and the analysis of fractal burgers hierarchy with the full effects of inhomogeneities of media. Rom. J. Phys. 2015; 60:324–43.
32. Syam M, Jaradat HM, Alquran M. A study on the two-mode coupled modified Korteweg-de Vries using the simplified bilinear and the trigonometric-function methods. Nonlinear Dyn. 2017; 90:1363–71. https://doi.org/10.1007/s11071-017-3732-6
33. Alquran M, Jaradat HM, Al-Shara S, Awawdeh F. A new simplified bilinear method for the N-soliton solutions for a generalized F mKdV equation with time-dependent variable coefficients. Int J Nonlinear Sci Numer Simul. 2015;16:259–69. https://doi.org/10.1515/ijnsns-2014-0023
34. Rezazadeh H, Vahidi J, Zafar A, Bekir A. The functional variable method to find new exact solutions of the nonlinear evolution equations with dual-power-law nonlinearity. Int J Nonlinear Sci Numer Simul. 2020; 21:249– 57. https://doi.org/10.1515/ijnsns-2019-0064
35. Yépez-Martínez H, Gómez-Aguilar JF. Fractional sub-equation method for Hirota-Satsuma-coupled KdV equation and coupled mKdV equation using theAtangana's conformable derivative. WavesRan Comp Med. 2019;29:678–93. https://doi.org/10.1080/17455030.2018.1464233
36. Yépez-Martínez H, Gómez-Aguilar JF, Baleanu D. Beta-derivative and sub equation method applied to the optical solitons in medium with parabolic law nonlinearity and high order dispersion. Optik. 2018;155:357– 65. https://doi.org/10.1016/j.ijleo.2017.10.104
37. Yépez-Martínez H, Gómez-Aguilar JF. M-derivative applied to the soliton solutions for the Lakshmanan-Porsezian-Daniel equation with dual-dispersion for optical fibers. Optical Quant Electron. 2019; 51:31. https://doi.org/10.1007/s11082-018-1740-5
38. Akinyemi L. Two improved techniques for the perturbed nonlinear Biswas-Milovic equation and its optical solutions. Optik-International Journal for Light and Electron Optics. 2021;243;167477.
39. Kumar S, Mann N, Kharbanda H, Inc M. Dynamical behavior of analytical soliton solutions, bifurcation analysis, and quasi-periodic

solution to the (2+1)-dimensional Konopelchenko–Dubrovsky (KD) system. Analysis and Mathematical Physics. 2023;13(3):40.
40. Kumar S, Mann N. A variety of newly formed soliton solutions and patterns of dynamic waveforms for the generalized complex coupled Schrödinger–Boussinesq equations. Optical and Quantum Electronics. 2023;55(8):723.
41. Kumar S, Rani S, Mann N. Diverse analytical wave solutions and dynamical behaviors of the new (2+1)-dimensional Sakovich equation emerging in fluid dynamics. The European Physical Journal Plus. 2022;137(11):1226.
42. Kumar S, Niwas M, Mann N. Abundant analytical closed-form solutions and various solitonic wave forms to the ZK-BBM and GZK-BBM equations in fluids and plasma physics. Partial Differential Equations in Applied Mathematics. 2021;4:100200.
43. Kumar S, Mann N. Abundant closed-form solutions of the (3+1)-dimensional Vakhnenko-Parkes equation describing the dynamics of various solitary waves in ocean engineering. Journal of Ocean Engineering and Science; 2022.
44. Rani S, Kumar S, Mann N. On the dynamics of optical soliton solutions, modulation stability, and various wave structures of a (2+1)-dimensional complex modified Korteweg-de-Vries equation using two integration mathematical methods. Optical and Quantum Electronics. 2023;55(8):731.
45. Nonlaopon K, Mann N, Kumar S, Rezaei S, Abdou MA. A variety of closed-form solutions, Painlevé analysis, and solitary wave profiles for modified KdV–Zakharov–Kuznetsov equation in (3+1)-dimensions. Results in Physics. 2022;36:105394.

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